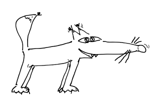
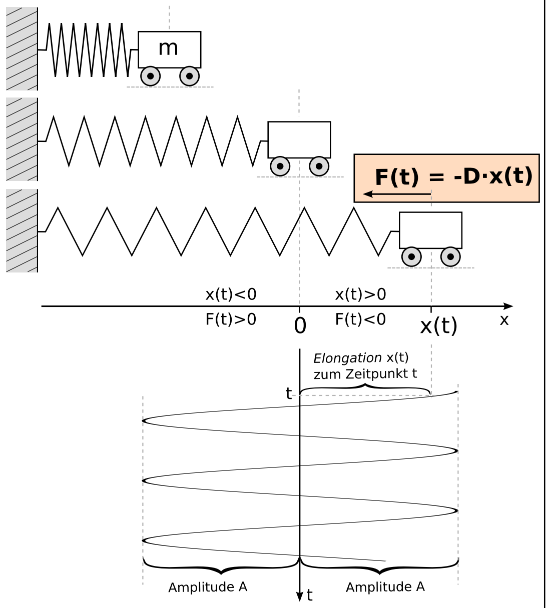
Harmonische Schwingung

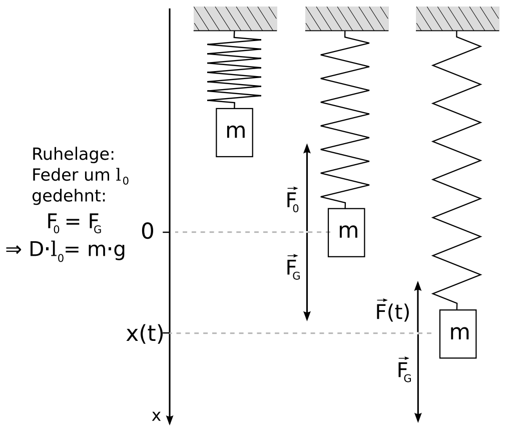
Gesucht: *lineares* Kraftgesetz der Form: $\boxed{\mathbf{F(t) = -D\cdot x(t)}}$

Rückstellkraft F(t) ist
- *linear abhängig* von der Auslenkung x(t)
- und ihr *entgegengesetzt*
- *Richtgröße* D ist *konstant*



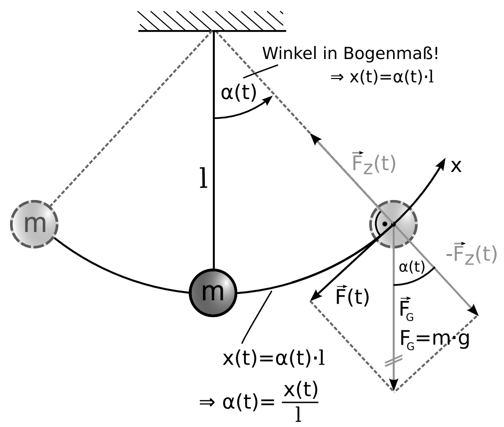


D ist Federkonstante



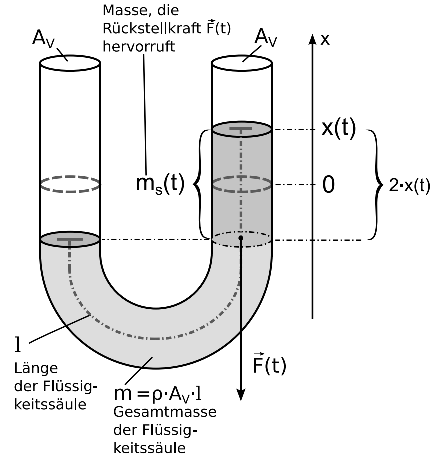
$$\mathbf{F(t)} = -D\cdot(l_0 + x(t)) + F_G = -\underbrace{D\cdot l_0}_{=m\cdot g} - D\cdot x(t) + m\cdot g = \mathbf{-D\cdot x(t)}$$

D ist Federkonstante



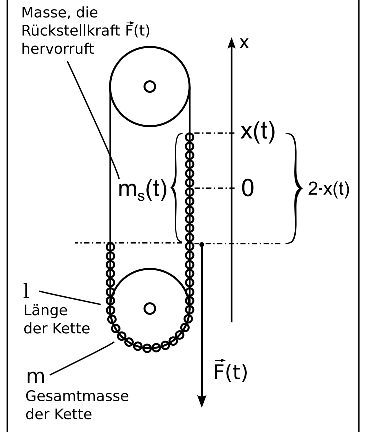
$$\mathbf{F(t)} = -F_G\cdot\sin(\alpha(t)) \approx -F_G\cdot\alpha(t) = -m\cdot g\cdot\frac{x(t)}{l} = -\underbrace{\frac{m\cdot g}{l}}_{=:D}\cdot x(t) = \mathbf{-D\cdot x(t)}$$

$$D = \frac{m\cdot g}{l}$$



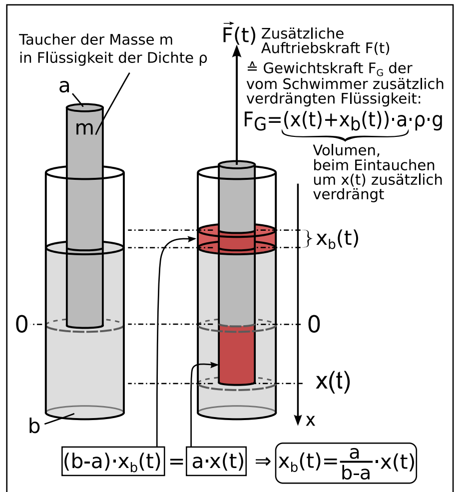
$$\mathbf{F(t)} = -m_S(t)\cdot g = -\rho\cdot V(t)\cdot g = -\rho\cdot A_V\cdot 2\cdot x(t)\cdot g = -\underbrace{2\cdot g\cdot\rho\cdot A_V}_{=:D}\cdot x(t) = \mathbf{-D\cdot x(t)}$$

$$D = 2\cdot g\cdot\rho\cdot A_V$$



$$\mathbf{F(t)} = -m_S(t)\cdot g = -m\cdot\frac{2\cdot x(t)}{l}\cdot g = -\underbrace{2\cdot g\cdot\frac{m}{l}}_{=:D}\cdot x(t) = \mathbf{-D\cdot x(t)}$$

$$D = 2\cdot g\cdot\frac{m}{l}$$



$$\mathbf{F(t)} = -(x(t)+x_b(t))\cdot a\cdot\rho\cdot g = -\left(x(t)+\frac{a}{b-a}\cdot x(t)\right)\cdot a\cdot\rho\cdot g = -x(t)\cdot\left(1+\frac{a}{b-a}\right)\cdot a\cdot\rho\cdot g = -\underbrace{\frac{b\cdot a}{b-a}\cdot\rho\cdot g}_{=:D}\cdot x(t) = \mathbf{-D\cdot x(t)}$$

$$D = \frac{b\cdot a}{b-a}\cdot\rho\cdot g$$

$\mathbf{F(t) = -D\cdot x(t)} \Rightarrow m\cdot\ddot{x}(t) = -D\cdot x(t)$ ← *Differenzialgleichung* der *harmonischen* Schwingung

$\Rightarrow \mathbf{x(t) = A\cdot\sin\left(\sqrt{\frac{D}{m}}\cdot t + \varphi_0\right)}$ ← Allgemeine Lösung der Differenzialgleichung

$\Rightarrow T = 2\pi\cdot\sqrt{\frac{m}{D}}$ und $\omega = \sqrt{\frac{D}{m}}$

$$x(t) = A\cdot\sin\left(\sqrt{\frac{D}{m}}\cdot t + \varphi_0\right) \qquad T = 2\pi\cdot\sqrt{\frac{m}{D}}$$

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$$x(t) = A\cdot\sin\left(\sqrt{\frac{g}{l}}\cdot t + \varphi_0\right) \qquad T = 2\pi\cdot\sqrt{\frac{l}{g}} \qquad \frac{D}{m} = \frac{\frac{m\cdot g}{l}}{m} = \frac{g}{l}$$

$$x(t) = A\cdot\sin\left(\sqrt{\frac{2g}{l}}\cdot t + \varphi_0\right) \qquad T = 2\pi\cdot\sqrt{\frac{l}{2g}} \qquad \frac{D}{m} = \frac{2\cdot g\cdot\rho\cdot A_V}{\rho\cdot A_V\cdot l} = \frac{2g}{l}$$

$$x(t) = A\cdot\sin\left(\sqrt{\frac{2g}{l}}\cdot t + \varphi_0\right) \qquad T = 2\pi\cdot\sqrt{\frac{l}{2g}} \qquad \frac{D}{m} = \frac{2\cdot g\cdot\frac{m}{l}}{m} = \frac{2g}{l}$$

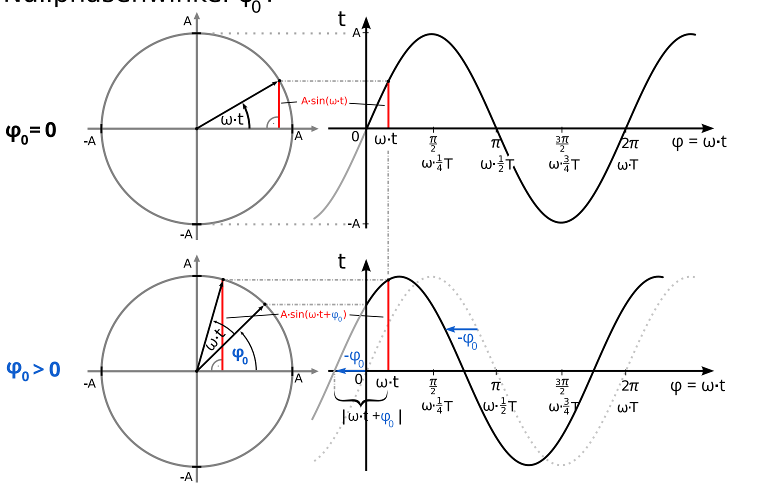
$$x(t) = A\cdot\sin\left(\sqrt{\frac{b\cdot a}{b-a}\cdot\frac{\rho\cdot g}{m}}\cdot t + \varphi_0\right) \qquad T = 2\pi\cdot\sqrt{\frac{b-a}{b\cdot a}\cdot\frac{m}{\rho\cdot g}}$$

Spezialfall: $b\to\infty$ (Schwimmer im Meer)

$$\lim_{b\to\infty}\left(\frac{b\cdot a}{b-a}\right) \overset{l'H}{=} \lim_{b\to\infty}\left(\frac{a}{1}\right) = a$$

$$\Rightarrow x(t) = A\cdot\sin\left(\sqrt{a\cdot\rho\cdot g\cdot m^{-1}}\cdot t + \varphi_0\right) \qquad T = 2\pi\cdot\sqrt{\frac{m}{a\cdot\rho\cdot g}}$$

Nullphasenwinkel φ_0:



Lineares Kraftgesetz der Form: $\mathbf{\underbrace{F(t)}} = \mathbf{-D\cdot x(t)}$

$= m\cdot a(t)$

$= m\cdot\ddot{x}(t)$

⇒ Differenzialgleichung der harmonischen Schwingung: $\mathbf{m\cdot\ddot{x}(t) = -D\cdot x(t)}$

$\Leftrightarrow \ddot{x}(t) = -\frac{D}{m}\cdot x(t)$

$\ddot{x}(t) = -\omega^2\cdot x(t)$, mit $\omega = \sqrt{\frac{D}{m}}$

⇒ Allgemeine Lösung: $\mathbf{x(t) = A\cdot\sin\left(\sqrt{\frac{D}{m}}\cdot t + \varphi_0\right)}$

Elongation (Weg): $\mathbf{x(t) = A\cdot\sin(\omega\cdot t+\varphi_0)}$

Geschwindigkeit: $\mathbf{v(t) = \dot{x}(t) = \omega\cdot A\cdot\cos(\omega\cdot t+\varphi_0)}$

Beschleunigung: $\mathbf{a(t) = \dot{v}(t) = \ddot{x}(t) = -\omega^2\cdot A\cdot\sin(\omega\cdot t+\varphi_0) = -\omega^2\cdot x(t)}$

Periode: $\mathbf{T = 2\pi\cdot\sqrt{\frac{m}{D}}}$

Amplitude: $\mathbf{A}$
(Betrag der maximalen Auslenkung)

Kreisfrequenz: $\mathbf{\omega = \sqrt{\frac{D}{m}}}$

Nullphasenwinkel: $\mathbf{\varphi_0}$
(auch: Phasenverschiebung(swinkel))

Phasenwinkel: $\mathbf{\omega\cdot t+\varphi_0}$